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On Characterizations of α^*-$T_{1/2}$ Spaces & Equivalence With Ap –irresolute & Ap -α-closed maps.

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Abstract

The purpose of the present paper is to highlight some characterizations of α^*-$T_{1/2}$ spaces. By using approximately irresolute as well as α-closed mappings, the equivalence of α^*-$T_{1/2}$ spaces has been introduced in the paper. Also some of the salient features of characterizations on α^*-$T_{1/2}$ spaces has been a part of the paper. At last we introduce ap-irresolute and ap-α-closed via the concept of gα-closed/ gα-open sets.

Key words: Topological spaces, generalized closed sets, gα-closed sets, α^*-$T_{1/2}$ space, irresolute maps, ap-α open maps.

1. Introduction

In the mathematical paper [3] O.Njastad introduced and defined an α-open/closed set. After the works of O.Njastad on α-open sets, various mathematicians turned their attention to the generalizations of various concepts in topology by considering semi-open, α-open sets.
The concept of g-closed [1], s-open [2] and α-open [3] sets has a significant role in the generalization of continuity in topological spaces. The modified form of these sets and generalized continuity were further developed by many mathematicians [4,5].

In this direction we introduce the concept of α-openness maps called ap-α open maps by using gα-closed sets and study some of their basic properties. This definition enables us to obtain conditions under which inverse maps preserve gα-open sets and also establish relationships between this map and other generalized forms of openness. Finally we characterize the class of α^*-$T_{1/2}$ spaces in terms of ap-α open maps.
Also, in this paper, we present a new generalization of irresoluteness called contra-α-irresolute. We define this last class of map by the requirement that the inverse image of each α-open set in the co-domain is α-closed in the domain. Finally, we also characterize the class of α^*-$T_{1/2}$ spaces in terms of ap-α-irresolute and ap-α-closed maps.

2. Preliminaries

As usual (X,τ), (Y,σ), and (Z,γ) represent nonempty topological spaces on which no separation axioms are assumed, unless otherwise mentioned. For a subset A of a space (X,τ), Cl(A), and Int(A) denote the closure of A and the interior of A, respectively.
We recall some definitions and properties essential in this paper.

Definition (2.1):
[I] A subset A of a topological space (X,τ) is called
(a) pre-open set[9] if $A \subset int(cl(A))$.
(b) semi –open set[2] if $A\subset cl(int (A))$.
(c) α-open[3] if $A\subset int(cl(int (A)))$.
(d) β-open [10] if $A\subset cl(int(cl (A)))$.
Here, the notions cl(A) and int(A) stand as the closure of A and the interior of A respectively. The complements of these sets are obviously the same type of closed sets.

Let (X,τ) be a topological space &$A\subseteq X$, then

(a) The family of all pre-open (resp. semi-open, α -open, β-open,) subsets of a space (X,τ) is denoted by $PO(X,\tau)$,($SO(X,\tau)$, $\alpha O(X,\tau)$,$\beta O(X,\tau)$).
(b) The family of all pre-closed(resp. semi-closed,α-closed,β-closed,) subsets of a space (X,τ) is denoted by.$PC(X,\tau)$($SC(X,\tau)$, $\alpha C(X,\tau)$,$\beta C(X,\tau)$).
(c)αint(A) stands for the α-interior of A, which is the union of all α-open subsets contained in A.
clearly, A = α int(A) means A is α-open.
(d) αcl(A) stands for the α -closure of A, which is the intersection of all α-closed subsets containing A.
clearly, A = αcl(A) means A is α-closed.

Definition (2.2): A subset A of a space (X, τ) is called

(a) g- closed[1] if $cl(A) \subseteq U$ whenever $A \subseteq U$ & U is open .
(b) αg-closed[7] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ & U is open .
(c) $g\alpha$-closed[7] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ & U is α-open.

The family of all g-closed , αg-closed,is denote by $gc(X, \tau)$,$\alpha gc(X, \tau)$.

Definition(2.3): In a topological space (X, τ), the following notions are well defined as :

(a) $\alpha D(X,\tau) = \{A: A\subset X$ and A is gα-closed in $(X,\tau)\}$.
(b) $\alpha cl^*(E) = \bigcap\{A : E \subset A(\in \alpha D(X,\tau))\}$
(c) $\alpha O(X,\tau)^* = \{B : \alpha cl^*(B^C) = B^C\}$.
(d) α^*-$T_{1/2}$ space if every $g\alpha$- closed set is α- closed.

3. characterizations of α^*-$T_{1/2}$ spaces.

Theorem 3.1 A topological space (X,τ) is a α^*-$T_{1/2}$ space if and only if $\alpha O(X,\tau) = \alpha O(X,\tau)^*$ holds.
Proof. Let (X,τ) be a topological space. Let E be the α-closed subset of (X,τ).
Necessity: As, $\alpha cl(A) = cl(A) = g\alpha cl(A)$.
i.e. the α-closed sets and the gα-closed sets coincide by the assumption, $\alpha cl(E) = \alpha cl^*(E)$ holds for every α-closed subset E of (X,τ).Hence , we have $\alpha O(X,\tau) = \alpha O(X,\tau)^*$.
Sufficiency: Let A be a α-closed set of (X,τ). Then, we have A = $\alpha cl^*(A)$
& by the accepted criteria $\alpha O(X,\tau) = \alpha O(X,\tau)^*$, we claim that $A^C \in \alpha O(X,\tau)$, which means that A is α-closed. Therefore (X, τ) fulfils the criteria for being α^*-$T_{1/2}$.

Theorem 3.2. A topological space (X, τ) is a α^* -$T_{1/2}$ space if and only if, for each x ∈X, {x} is α-open or α-closed.

Proof. Let topological space (X, τ) be a α^* -$T_{1/2}$ space.

Necessity: Let topological space (X, τ) be a α^* -$T_{1/2}$ space. Let us Suppose that for some $x \in X$; $\{x\}$ is not α-closed. Since X is the only α-open set containing $\{x\}^C$, the set $\{x\}^C$ is gα-closed [definition(2.3)(d)] and so it is α-closed in the α^* -$T_{1/2}$ space (X, τ). Therefore {x} is α-open.

Sufficiency: Since $\alpha O(X,\tau) \subseteq \alpha O(X,\tau)^*$ holds, it is enough to prove that $\alpha O(X,\tau)^* \subseteq \alpha O(X,\tau)$. Let $E \subseteq \alpha O(X, \tau)$. Suppose that $E \notin \alpha O(X, \tau)$. Then, $\alpha cl^*(E^C) = E^C$ and $\alpha cl(E^C) \neq E^C$ hold. There exists a point x of X such that $x \in \alpha Cl(E^C)$ and $x \notin E^C$ $(= \alpha cl^*(E^C))$. Since $x \notin \alpha Cl\ (E^C)$ there exists a gα-closed set A such that $x \notin A$ and $A \supset E^C$ By the hypothesis, the singleton {x} is α-open or α-closed.

Case 1. {x} is α-open: Since$\{x\}^C$ is a α-closed set with $E^C \subset \{x\}^C$, we have $\alpha Cl(E^C) \subset \{x\}^C$ i.e. $x \notin \alpha Cl\ (E^C)$ .This contradicts the fact that $x \in \alpha Cl(E^C)$. E∈ $\alpha O(X, \tau)$.

Case 2. {x} is α-closed: Since$\{x\}^C$ is a α -open set containing the gα-closed set $A(\supset E^C)$, we have$\{x\}^C \supset \alpha Cl(A) \supset \alpha Cl(E^C)$ Therefore $x \notin \alpha Cl\ (E^C)$ This is a contradiction. Therefore E∈ $\alpha O(X, \tau)$.

Hence in both cases, we have E∈ $\alpha O(X, \tau)$, i.e., $\alpha O(X,\tau)^* \subseteq \alpha O(X, \tau)$.

$\therefore \alpha O(X,\tau) = \alpha O(X,\tau)^*$ using theorem (3.1), it follows that (X, τ) is a α^* -$T_{1/2}$

Theorem 3.3. A topological space (X, τ) is a α^* -$T_{1/2}$ space if and only if and only if, every subset of X is the intersection of all α-open sets and all α-closed sets containing it.

Proof. **Necessity:** Let topological space (X, τ) is a α^* -$T_{1/2}$ space with $B \subset X$ arbitrary. Then B = $\bigcap \{\{x\}^C ; x \notin B\}$is an intersection of α-open sets and α-closed sets by Theorem 3.2. So the necessity follows.

Sufficiency: For each $x \in X, \{x\}^C$ is the intersection of all α-open sets and all α-closed sets containing it. Thus $\{x\}^C$ is either α-open or α-closed and hence X is α^* - $T_{1/2}$ space .

In the following definitions & theorems, we give a characterization of a class of topological space called α^* -$T_{1/2}$ space by using the concepts of ap-irresolute maps and ap-α-closed maps.

Definition (3.1): Approximately -α-irresolute:

A mapping $f:(X,\tau) \to (Y,\sigma)$ from one topological space (X,τ) to another topological space(Y,σ) is said to be approximately irresolute(ap-α-irresolute) if $\alpha cl(F) \subseteq f^{-1}(O)$ whenever O is a α-open subset of (Y,σ),F is a gα-closed subset of(X,τ), $F \subseteq f^{-1}(O)$

Definition (3.2):Approximately-α-closed Map:

A mapping $f:(X,\tau) \to (Y,\sigma)$ from one topological space (X,T) to another topological space(Y,σ) is said to be approximately α-closed(Ap-α-closed) if f(B) ⊆ αint (A) whenever A is a gα-open subset of (Y,σ), B is a α-closed subset of (X,τ), and f(B)⊆ A.

Theorem 3.4. For a topological space (X,τ) the following are equivalent:

(i) (X, τ) is a α^* -$T_{1/2}$ space.

(ii) For every space (Y,σ) and every map $f:(X,\tau)\to(Y,\sigma)$, f is ap-irresolute.

Proof. (i)⟶(ii). Let F be a gα-closed subset of (X,τ) and suppose that $F\subseteq f^{-1}(O)$ where $O\in\alpha O(Y,\sigma)$. Since (X, τ) is a α^* -$T_{1/2}$ space, F is α-closed (i.e., F = αCl(F)). Therefore $\alpha cl(F)\subseteq f^{-1}(O)$. Then f is ap irresolute.

(ii)→(i). Let B be a gα-closed subset of(X, τ) and let Y be the set X with the topology σ = {φ,B,Y}. Finally let $f:(X,\tau)\to(Y,\sigma)$ be the identity map.

By assumption f is ap-α-irresolute. Since B is gα-closed in(X, τ) and α-open in (Y,σ_) and $B\subseteq f^{-1}(B)$, it follows that $\alpha cl(B)\subseteq f^{-1}(B)=B$. Hence B is α-closed in(X, τ) and therefore (X, τ)is a α^* -$T_{1/2}$ space.

Corollary(3.1): For a topological space(X, τ) the following are equivalent:

(i) (Y,σ) is a α^* -$T_{1/2}$ space.

(ii) For every space (X, τ) and every map $f:(X,\tau)\to(Y,\sigma)$,f is ap-α-closed.

Proof. Analogous to Theorem 3.4 making the necessary changes.

4. Approximately α-open/closed maps.

Let $f:(X,\tau)\to(Y,\sigma)$ be a map from one topological space (X,τ) to another topological space(Y,σ) .

Definition (4.1):

(a) A map $f:(X,\tau)\to(Y,\sigma)$ is said to be approximately -open (or ap-α-open) if, αCl(B) ⊆ f(A) whenever *B* is a gα-closed subset of (*Y*, σ),A is a α-open subset of (*X*,τ) and B⊆f(A).

(b)A map $f:(X,\tau)\to(Y,\sigma)$ is said to be approximately- α-closed (or ap-α-closed) if, *f*(*B*) ⊆ αInt(A) whenever *A* is a gα-open subset of (Y,σ), B is a α-closed subset of (*X*,τ) and f(B) ⊆ A.

(c) A map $f:(X,\tau)\to(Y,\sigma)$ is called α-closed (resp. α-open) if, for every α-closed (resp. α-open) set *B* of(X,τ) ; *f*(*B*) is α-closed (resp. α-open) in (Y,σ).

Observations:

- ap-α-openness and ap-α-closedness are equivalent if the map is bijective. Clearly α-open maps are ap-α-open, but not conversely.

The following example shows that the converse implications do not hold.

Example 4.2. Let *X* = {a,b} be the Sierpinski space with the topology (X,τ) ={φ,{a}, X} Let *f* : *X* →*X* be denoted by *f*(*a*) = *b* and *f*(*b*) = *a:* Since the image of every α-open set is α-closed, then *f* is ap-α-open. However {a} is α-open in (X,τ)) but f({a}) is not α-open in (*Y*,σ). Therefore *f* is not ap-α-open.

Theorem 4.3. If $f:(X,\tau)\to(Y,\sigma)$ is ap-α-open then, f(O) ∈ αC(Y,σ) for every α-open subset O of (X,τ).

Proof. Let B ⊆ f(A) ,where *A* is a α-open subset of (*X*,τ) and *B* is a gα-closed subset of (*Y*,σ). Therefore αCl(B) ⊆αCl(f(A)) = f(A). Thus *f* is ap-α-open.

Theorem 4.4. Let $f:(X,\tau)\to(Y,\sigma)$ be a map from a topological space(X,τ) to another topological space(Y,σ) . If the α-open and α-closed sets of (*Y*, σ) coincide, then f is ap-α-open if and only if, f(A)∈ αC(Y,σ) for every α-open subset A of (X,τ).

Proof. Assume *f* is ap-α-open. Let *A* be an arbitrary subset of (Y,σ) such that $A \subseteq Q$ where $Q \in \alpha O(Y,\sigma)$: Then by hypothesis $\alpha Cl(A) \subseteq \alpha Cl(Q) = Q$: Therefore all subset of (Y,σ) are gα-closed (and hence all are gα-open). So for any O ∈ αO(X,τ),f(O) is gα-closed in (Y,σ). Since *f* is ap-α-open αCl(f(O)) ⊆ f(O). Therefore αCl(f(O)) = f(O) , i.e., f(O) is α-closed in (Y,σ):

Theorem 4.5. If $f:(X,\tau)\to(Y,\sigma)$ is surjective irresolute and ap-α-open, then $f^{-1}(A)$ is gα-open whenever A is gα-open subset of (Y,σ),

Proof. Let *A* be a gα-open subset of (Y,σ): Let us Suppose that

$F \subseteq f^{-1}(A)$, where $F \in \alpha C(X,\tau)$.

$\Rightarrow f^{-1}(A^C) \subseteq F^C$ (Taking complements both side).

Since, *f* is an ap-α-open and $\alpha Int(A) = A \cap int(Cl(Int(A))$

$\& \ \alpha Cl(A) = A \cup Int(Cl(intA))$,

then $(\alpha Int(A))^C = \alpha Cl(A^C) \subseteq f(F^C)$.

$\Rightarrow (f^{-1}(\alpha Int(A)))^C \subseteq F^C$

$\Rightarrow (F \subseteq F^{-1}(\alpha Int(A))$: (Since f is irresolute $F^{-1}(\alpha Int(A))$ is α-open).

$\Rightarrow F \subseteq F^{-1}(\alpha Int(A)) = \alpha Int(F^{-1}(\alpha Int(A))) \subseteq \alpha Int(F^{-1}(A))$.

$\Rightarrow F^{-1}(A)$ is gα-open in(X,τ).

Hence, the theorem.

Theorem 4.6. Let (Y, σ) be a topological space. Then the following statements are equivalent.

(i) (Y,σ) is a α^* -$T_{1/2}$ space,

(ii) For every space (X,τ) and every map $f:(X,\tau)\to(Y,\sigma)$ is ap-α-open.

Proof. (i)→ (ii) : Let *B* be a gα-closed subset of (Y,σ) and suppose that $B \subseteq f(A)$ where A ∈ αO(X,τ): Since (Y,σ) is a α^*-$T_{1/2}$ space, *B* is α-closed (i.e., $B = \alpha Cl(B)$): Therefore $\alpha Cl(B) \subseteq f(A)$: Then *f* is ap-α-open.

(ii) → (i) : Let B be a αg-closed subset of (Y,σ) and let X be the set Y with the topology τ = {φ,B,X} Finally let $f:(X,\tau)\to(Y,\sigma)$ be the identity map. By assumption *f* is ap-α-open. Since *B* is αg-closed in (X,τ) and α-open in (X,τ) and $B \subseteq f(B)$; it follows that $\alpha Cl(B) \subseteq f(B) = B$: Hence *B* is α-closed in (Y,σ). Therefore (Y,σ) is a α^* -$T_{1/2}$ space .

Theorem 4.7. Let $f:(X,\tau)\to(Y,\sigma)$ be two maps $g:(Y,\sigma)\to(Z,\gamma)$ such that g o f = $f:(X,\tau)\to(Z,\gamma)$ Then

(i) g o f is ap-α-open, if f is α-open and g is ap-α-open.

(ii) g o f is ap-α-open, if f is ap-α-open and g is bijective α-closed and gα-irresolute.

Proof. In order to prove the statement (i), suppose *A* is an arbitrary α-open subset in (X,τ) and *B* a gα-closed subset of (Z,γ) for which $B \subseteq$ g o f(A). Then $f(A)$ is α-open in (Y,σ) because *f* is α-open. Since *g* is ap-α-open, $\alpha Cl(B) \subseteq g(f(A))$: This implies that g o f is ap-α-open.

In order to prove the statement (ii), suppose *A* is an arbitrary α-open subset of(X,τ) and *B* a gα-closed subset of (Z,γ) for which $B \subseteq$ g o f(A)):

Hence $g^{-1}(B) \subseteq f(A)$: Then $\alpha Cl(g^{-1}(A)) \subseteq f(A)$ because $g^{-1}(B)$ is gα-closed and *f* is ap-α-open. Hence we have,

$\alpha Cl(B) \subseteq \alpha Cl(gg^{-1}(B)) \subseteq g(\alpha Cl(g^{-1}(B))) \subseteq g(f(A)) = (g\ of)(A)$:

This implies that *g o f* is ap-α-open.

Conclusions:

As Generalized closeness is one of the most important useful and fundamental concept in topology so its structural properties can also be emphasized in the form of α^*-$T_{1/2}$, α-closed space which open a new horizon in the space of Mathematics through this paper. The structures mentioned in the paper have wide applications and are bound to reflect ap-irresoluteness; ap-α-open/closed sets etc.

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